Steve **Awodey—***Intensionality, Invariance, and Univalence*

What does a mathematical proposition mean? Under one standard account, all true mathematical statements mean the same thing, namely True. A more meaningful account is provided by the Propositions-As-Types conception of type theory, according to which the meaning of a proposition is its collection of proofs. The new system of Homotopy Type Theory provides a further refinement: The meaning of a proposition is the homotopy type of its proofs. A homotopy type may be seen as an infinite-dimensional structure, consisting of objects, isomorphisms, isomorphisms of isomorphisms, etc. Such structures represent systems of objects together with all of their higher symmetries. The language of Martin-Löf type theory is an invariant of all such higher symmetries, a fact which is enshrined in the celebrated Principle of Univalence.

Bob **Coecke**—*Physics from Compositional Logic*

The point put forward in this talk is that quantum physics can be entirely described in terms of compositional logic, where the latter refers to the low-dimensional topological reasoning mechanisms of string diagrams. First we review some recent results in categorical quantum mechanics, most notably, a completeness theorem that shows that all equational reasoning in Hilbert space can be done in terms of string diagrams. We then substantiate our claim that string diagram reasoning indeed has to be considered as a `logic', by first showing that the same string diagrams govern natural language, not just grammar but also meaning interaction. This can traced back to what could be called `the fundamental logic of reality', in that linguistic structures are a reflections of all that is happening in reality, which we substantiate by providing functors between language and spatial perception, as well as other sensory/cognitive modes.

Michał **Eckstein—***The Experiment Paradox*

The foundations of quantum mechanics are haunted by the notorious "measurement problem" and the consequent indeterminism of the measurement outcomes. I will show that it is in fact an instance of the experiment paradox lurking in the very core of the scientific method. Concretely, any experiment performed on a physical system is—by necessity—invasive and thus establishes inevitable limits to the accuracy of any mathematical model. In consequence, the foundations of natural sciences turn out to face similar limiting problems as the foundations of mathematics. In particular, the "random" events at the experimental input play a similar role as the axioms in mathematics. The experiment paradox suggests that the classical logic is unable to provide a unified picture of natural phenomena. The talk is based on a recent preprint with Pawel Horodecki: https://arxiv.org/abs/1904.04117

Michał **Heller**—*The Robot Mind and Categorical Logic*

"The unreasonable effectiveness of mathematics in the natural sciences'" consists in the fact (corroborated by the whole of history of science) that some formal mathematical structures (a "syntax") are somehow implemented (acquire their "semantics") in the physical world. The interaction between syntax and its semantics is thoroughly studied in categorical logic. It is well known that for each formal theory, it is possible to construct a category providing a semantics for this theory and, *vice versa*, for each category it is possible to construct a formal theory, the syntax of which corresponds to the internal logic of this category. The interaction between a category of formal theories and a category of corresponding categories is modeled by the pair of adjoint functors, called Lan and Syn.

We propose a model of the robot brain, the neurons of which are represented by categories and transmission processes between neurons by functors between these categories. We postulate the existence of functors, analogous to Lan and Syn, that would model the interaction between the BRAIN of the robot, i.e. the category of all neuron-theories, and its MIND, i.e. the category of corresponding categories. If we ascribe to these functors "the unreasonable effectiveness" typical for mathematical structures, we might claim that the robot BRAIN generates its MIND, and the robot MIND shapes its BRAIN.

The robot parable is intended to emphasize the toy character of the model.

Ryszard **Kostecki**—*Two Layers of Inference*

I will discuss how two structural layers of physical theories, ontic and epistemic, reflect two different forms of inference, deductive and inductive, respectively. On mathematical side, I will focus on topos theoretic models of analysis and geometric structure of state spaces of operator algebras. On physical side, I will focus on general relativity and quantum theory.

Jerzy **Król**, Torsten **Asselmeyer-Maluga**—*Differentiability, Logic and Physics*

We discuss differentiable smoothness structures on $\boldsymbol{R}^4$ from three different categorical perspectives. The first one relies on considering open atlases on $\boldsymbol{R}^4$ with certain (not all) of its local charts residing in a smooth topos. Thus exotic smooth functions on $\boldsymbol{R}^4$ are finelly approached without any use of Casson handles and handle decompositions. The second approach takes into account entire space of all smoothness structures on $\boldsymbol{R}^4$. Forcing extensions naturally order the structures and show a way towards new smooth invariants of exotic R^4's. The third approach shows how logical structure of quantum mechanics enforces exotic smoothness at large cosmological scales. Finally, we present recent result showing that the very tiny value of the cosmological constant can be understood as a topological invariant derived from certain small exotic R^4. Is varying logic necessary to solve important physical problems?

(1) Asselmeyer-Maluga, T., Król, J. How to obtain a cosmological constant from small exotic $\boldsymbol{R}^4$. *Physics of the Dark Universe* 2018, 19, 66-77.
(2) Etesi, G. Strong cosmic censorship and topology change in four dimensional gravity, arXiv:1905.03952
(3) Król, J. Background Independence in Quantum Gravity and Forcing Constructions. *Foundations of Physics* 200434, No. 3, 361-403.
(4) Król, J. Model and set-theoretic aspects of smoothness structures on $\boldsymbol{R}^4$, in *At the Frontier of Spacetime*, Asselmeyer-Maluga, T. ed.; Fundamental Theories of Physics vol 183, Springer: Switzerland; 2016; pp. 217-240.
(5) Moerdijk, I., Reyes, G. E. *Models for Smooth Infinitesimal Analysis*; Springer Science + Business Media: New York, USA, 1991.

Zbigniew **Król**—*Ontology and Logic*

The relationships between ontology and logic as well as the role of logic in ontological research are considered. In particular, the influence and role of logic in formal ontology, ontology of physical theory and ontology of mathematics will be discussed. The argument is for a positive answer to the question whether the choice and possible change of logic and the use of category theory tools are necessary and needed in modern ontology.

Wiesław **Kubiś**—*Generic Mathematical Structures*

A mathematical object can be called "generic" if it appears, up to isomorphism, with probability one as the result of a natural stochastic process. Instead of probability, one may its topological counterpart, using the Baire category theorem. Yet another option is using a natural infinite game for two players, declaring an object *U* "generic" if one of the players has a suitable winning strategy leading to the isomorphic copy of *U*.

The story of generic mathematical structures goes back to Cantor, who was the first to identify the set of rational numbers as the generic countable linearly ordered set. About half a century later, Fraïssé developed an abstract theory of universal homogeneous structures (nowadays called "Fraïssé limits") which until recent years was viewed as a part of model theory.

As it happens, Fraïssé limits are particular cases of generic mathematical objects which can be found in several branches of mathematics, starting from model theory, algebra, functional analysis, and geometric topology. We will try to explain why pure and enriched category theory is the suitable language and framework for studying these objects.

Marek **Kuś**—*No-Signaling in Categorical Formulation*

Exploring a particular examples of "postquantum" concepts I will try to discuss ontological implications of category theory when applied to physical theories. The ontological status of such fundamental elements of physical reality, as positions, momenta, angular momenta, etc. have radically different ontological status in classical and theories. Whereas they are intrinsic and objective properties of a classical physical system, it is not so in quantum theory. From a purely physical point of view this is not a danger. Ultimately, physics is an experimental science. It can and should answer experimental questions about outcomes of various measurements. Such an approach clearly puts more emphasis on the epistemology, moving apart, or even totally discarding ontological issues.

The "categorization" can be looked upon as a kind of restoration of underlying ontology of classical, quantum and no-signaling theories in the phase space. It can also can be treated as a kind of an "ontology shift", however, the shift does not enrich the ontology, as it is the case in mathematics, but rather impoverishes it, e.g., by denying objective existence, or at least a primary ontological character, to some properties like positions and momenta. This is, probably, the price we pay for an ontological unification of the mentioned physical theories in the frames of the category theory.

Radosław **Kycia**—*Yes. Information is physical—Landauer's principle as a special case of Galois connection*

I will present categorification of entropy and the second law of thermodynamics. Then I will show that the Landauer's principle that associates information with physical realization of it is a special case of a well-known Galois connection—an example of adjointness. Finally, I will present applications to mathematics, physics, DNA computing and the meaning of life. The talk is based on [1-2].

[1] R.A. Kycia, Landauer's Principle as a Special Case of Galois Connection, Entropy 2018, 20(12), 971; https://doi.org/10.3390/e20120971

[2] R.A. Kycia, Entropy in Themodynamics: from Foliation to Categorization, arXiv:1908.07583 [math-ph]

Shahn **Majid**—*Riemannian Geometry on Boolean Algebras*

Recent work with A. Pachol on `digital' quantum Riemannian geometry over the field {0,1} of two elements includes the case of the Boolean algebra of subsets of a set of three elements. This turns out to have a natural differential structure with exterior algebra resembling a 2-manifold, a unique Riemannian metric with respect to this, and four quantum Levi-Civita connections for the metric, of which one is flat and three are Ricci flat. We exhibit this geometry and some generalisations with a view to exploring gravity in the context of propositional logic.

Jean-Pierre **Marquis**—*Bourbaki, Categories and Structuralism*

Bourbaki's structuralism is usually ignored or at best dismissed by philosophers of mathematics. In this talk, I will revisit Bourbaki's structuralism and my main claim is that Bourbaki's structuralism has been misunderstood and that it can be extended to include more abstract concepts, e.g. categories, in a coherent way. I will first briefly present the historical development of Bourbaki's structuralism, how it should be understood and why it is philosophically relevant. I will then show how the appearance of categories, functors and natural transformations got the best of Bourbaki's structuralism. Finally, I will sketch how the work of Grothendieck, Lawvere and others can be seen as the direct extension of Bourbaki's structuralism and how it is still philosophically relevant.

Colin **McLarty**—*Mathematics as Love of Wisdom*

While Saunders Mac Lane knew very well the difference between a philosophic argument and a mathematical proof, and he was all too familiar with the different intellectual cultures of mathematics departments and philosophy departments, he never believed mathematics could exist apart from the whole love of wisdom—that is from philosophy.

Zbigniew **Semadeni**—*Creating New Concepts in Mathematics: Freedom and Limitations*

The celebrated dictum of Georg Cantor "The very essence of mathematics lies precisely in its freedom" (1879) expressed the idea that in mathematics one can freely introduce new notions (which may, however, be abandoned if found unfruitful or inconvenient). Cantor opposed the efforts of Kronecker who demanded to restrict the objects of mathematics to constructive, finitistic ones only.

It is clear that the freedom of mathematics is limited by logical constraints. A mathematician trying to prove a theorem knows the feeling of an invisible wall which blocks the intended argument. Moreover, new concepts must be consistent with earlier ones and must not lead to contradiction or ambiguity.

Jan Łukasiewicz (1910) distinguished *constructive notions* from *reconstructive* ones, i.e., empirical. He referred (with reservation) to Dedekind's statement that the constructed notions are free creation of the human mind and pointed out that a consequence of our "creation" of those notions is the spontaneous emergence of countless relations which no more depend on our will.

The purpose of the talk is to discuss the question of freedom vs. limitation in the development of mathematics combined some traditional philosophical issues. We will look for criteria to distinguish between concepts which are natural follow-ups of the previous ones and concepts which were opening new ways of thought and reasoning, with particular attention to the rise of category theory.

Bartłomiej **Skowron**—*Was Saunders Mac Lane a Platonic?*

Saunders Mac Lane, on the basis of a comprehensive examination of contemporary mathematics, claimed that mathematics develops by extracting nebulous ideas from facts and then by codifying these ideas in mathematical forms. Mathematics is a protean because one an the same idea can have many realizations. Mac Lane firmly stated that his position had nothing to do with mythical Platonism. I ask what Mac Lane's ideas are if they are not Platonic? For this purpose, I use Roman Ingarden's phenomenological ontology and show that Mac Lane's ideas, contrary to his claims, belong to the sphere of ideal being. Mac Lane's forms, in turn, are intentional objects studied by phenomenologists. I present the richness of Mac Lane's ontology of mathematics, claiming that Mac Lane was also a great theorist of ideas.

Mariusz **Stopa**—*Is There Any Place for Paraconsistent Logic in (Co-)Toposes?*

Category theory and especially topos theory have changed the way we think about the role of logic in mathematics and through mathematics perhaps also in physics. It is a common knowledge that toposes are intrinsically connected to intuitionistic logic (or more precisely intermediate logic). However, recently there appeared several works concerning so-called complement-toposes (co-toposes) connected to, supposedly, paraconsistent logic (cf. i.a. Morensen *Inconsistent Mathematics* (1995), Estrada-Gonzalez *Complement-Topoi and Dual Intuitionistic Logic* (2010)). If that is true these new categorical structures could expand the possible inner connections between logic and category theory. Yet, it seems for me that at least some aspects of co-toposes are inappropriately defined or introduced, at least without any further comment.

In my talk, I would like to contribute to the critical analysis of the concept of co-topos. I shall investigate the question of possible interpretations of a distinguished by Ω-axiom arrow $1 \to \Omega$. Is its meaning as truth imposed by the very structure of the topos or is it open to different interpretations (e.g. as falsity)? If it can be interpreted otherwise than true, what would be the consequences of such an interpretation? I shall try to face these questions and offer some examples. Among others I shall explore the use of Yoneda lemma in connection with representability of the Sub functor and determination of subobject classifier with its distinguished arrow $1 \to \Omega$. If time permits I would like to mention bi-Heyting toposes, known in the literature, for which all the algebras of subobjects are also co-Heyting (and thus bi-Heyting) and/or importance of (co-)sheaves for the question of the status of co-toposes.

Marek **Woszczek**—*Quantum Contextuality as a Topological Property, and the Ontology of Potentiality*

Contextuality is a fundamental, irreducible physical property of quantum systems, which is a direct consequence of the mathematical structure of the quantum algebras of observables, as expressed by the Kochen–Specker Theorem. In general terms, it is an impossibility of the consistent, global assigning the pre-existing {0,1}-values to all possible physical observables on a quantum system for dim>2, which is a serious deviation from the classical-mechanical case where such assignments are always possible. Recently, there has been a growing body of experimental tests of quantum contextuality, both state-dependent and state-independent, on diverse physical systems such as single photonic qutrits, neutrons, ion trapped systems or nitrogen-vacancy systems. I shall stress that the spacelike nonlocality for compound entangled systems is just a special case of contextuality, and the latter manifests in temporal sequences of events for single simple systems, such as qutrits, when nonlocal behavior is a priori absent (with no Bob and Alice). There is an ongoing controversy concerning the interpretation of these noncontextuality inequality tests, and it shall be suggested that there is some purely philosophical issue at the core of the debate.

In the first part of the talk I shall argue that ontic contextuality is the generic characteristics of quantum systems, and it is their topological property which can be studied in the sheaf-theoretic framework in terms of the obstructions to global sections (concatenations) of the quantum {0,1}-valuation sheafs. That topological property can manifest in many different ways, but its 'pure' manifestation is strictly temporal, in the case of single quantum systems evolving in time and the cyclic non-demolition measurements on them, as in the standard noncontextuality inequality violation tests. I shall argue for its overtly ontological interpretation due to its physical, in particular thermodynamic, consequences for quantum information processing in nature. The impossibility of reintroducing the measurements of pre-existing properties calls the very ideas of a mechanical 'state' pertaining to a system as well as the 'measurement' (detection) into question as classical artifacts, however it is far from clear what further ontological lessons should be drawn from it.

In the second part of the talk I shall clarify those issues by discussing in detail how is contextuality incorporated into two distinct approaches to the quantum mechanics: deterministic, like the hidden-variable Bohmian mechanics without any collapse, and indeterministic, like the more standard approaches with the irreducibly random 'collapse'. I shall indicate what sort of ontological commitments is required by those diverging approaches regarding contextuality, and argue that if there is some ontological lesson from such a strong contextuality of the quantum information processing, it is rather that one should take both physical reality of potentiality and the ontic randomness seriously. The fundamental metaphysical convictions such as (in)determinism cannot be falsified, but some particular interpretations of contextuality have physical consequences and are also logically connected to other physical assumptions, which may be together evaluated and, in some cases, even put into experimental test. I shall discuss the common philosophical ground for preferring the hidden variable models of quantum contextuality as a hidden and dubious metaphysical assumption that everything real is (and can only be) actual, hence there are no real (ontic) possibilities in the world. However, those possibilities, if real, cannot be captured by the classical probability theory, hence the open space for some more constructive approaches in metaphysics.